TEACHER GUIDE

Space Careers Wayfinder

Getting off the ground

Background

A group made up of some of the world's leading space agencies, including the Australian Space Agency are working with NASA on a mission to return to the moon. The Artemis Program not only intends to put humans on the surface of the moon, but the plan for the program includes a long-term lunar presence. This could eventually serve as a steppingstone for future missions to Mars.

To get any space craft from Earth to the Moon requires a huge collaborative effort. From construction of the craft, to launch into space, every component and every element of the process is subject to meticulous quality and safety controls.

The Brief

The education program manager of a space education centre has put together a student activity for use in senior school. The activity is based around the mission to develop a base on the moon and some of the challenges associated with the mission. The manager is now looking to trial the activity with students from Year 9 and Year 10. Completing the activity will provide her with valuable feedback and allow her to modify the content where necessary.

The Task

Check through the activity developed by the space education program manager, checking their calculations, and working through their ideas. Complete the tasks marked ☆

The Student Activity

Earth's gravity is what prevents us from 'floating off' the planet. For any object to escape the pull from Earth's gravity and explore deep space scientists have calculated it would need to reach a velocity of 11.2 km/s or 40 243 km/h.

✭ Use the following formula and information to prove, or disprove the above velocity value:

Formula: $Ve = \sqrt{\frac{2GM}{r}}$

Ve (m/s) = Escape Velocity from Earth

$G = 6.674 \times 10^{-11}\ m^3kg^{-1}s^{-2}$ (Newton's universal constant of gravity)

$M = 5.972 \times 10^{24}$ kg (mass of planet leaving from – [Earth])

$r = 6.378 \times 10^{6}$ m (radius of planet – [Earth])

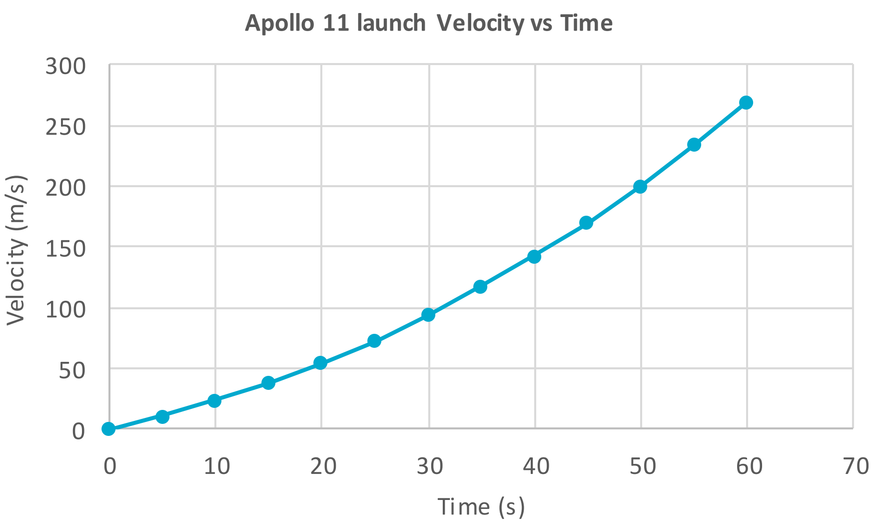
$$Ve = \sqrt{\frac{2(6.674 \times 10^{-11})(5.972 \times 10^{24})}{6.378 \times 10^{6}}}$$

Ve = 11 180 m/s (or 40 247 km/h)

The gravitational constant g is a measure of acceleration due to gravity. The force we experience on the surface of the Earth is equal to 1g. Astronauts aboard the Apollo 11 mission to the moon experienced up to 4.5 g, during take-off.

✭ Using the Apollo 11 launch footage (https://www.youtube.com/watch?v=Vg9WolsXbIA) slowed to 0.5 normal speed. Record the time and velocity of the launch every 5 seconds from take-off (0 seconds) to 60 seconds, plot your recordings and answer the following:

Time (s)	Velocity (m/s)
0	0
5	11
10	24
15	38
20	55
25	73
30	94
35	118
40	143
45	170
50	200
55	234
60	269



NOTE student recording are likely to vary from the above, depending on actual recording time.

1. What can you say about Apollo 11 acceleration for the first 60 seconds?
   Acceleration is increasing (this can be determined from the gradient or slope of the graph)
2. What is the rate of acceleration between 50 seconds and 60 seconds?
   Around 6.9 m/s^2
3. Using the g Force formula (below) what would be the approximate g force experienced by the astronauts for the 10 seconds between 50 seconds and 60 seconds after launch? (NOTE – g force would need to be greater than 1 g or the rocket wouldn't be able to leave the surface)

$$\text{g force} = \frac{V_1 - V_0}{tg}$$

Where: v_1 = velocity at 60 seconds
v_0 = velocity at 50 seconds
t = time
g = acceleration due to gravity (9.81 m/s^2)

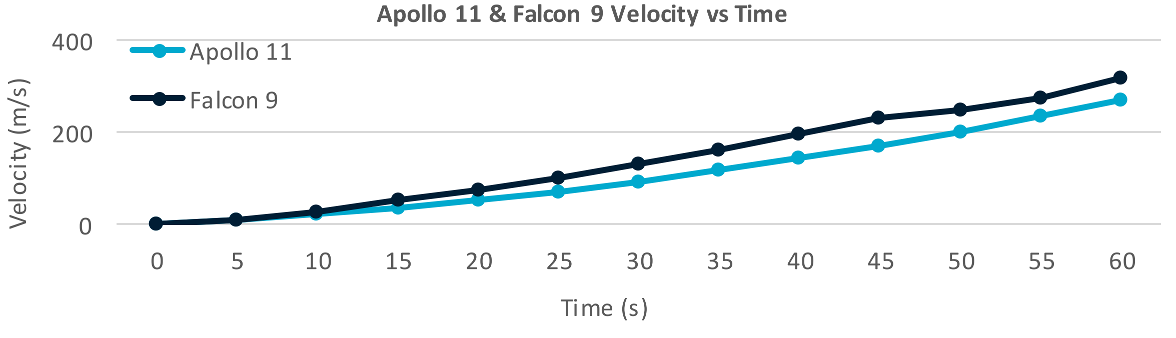
g force due to thrust = (269 - 200)/(10 x 9.81)
= 0.703 g
g force total = 0.703 g + 1 g (due to the gravity of the Earth)
= 1.7 g (1 d.p.)

SpaceX is a spacecraft manufacturer, launcher, and a satellite communications corporation based in the US. At the core of their spacecraft fleet is the Falcon 9 rocket. Unlike previous space craft, the Falcon 9 is a reusable rocket, and in January 2023 SpaceX used one of their Falcon 9 rockets on its 15th mission.

The approximate velocity data below was collected from the live broadcast of the January 2023 launch [1]

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55	60
≈ Velocity (m/s)	0	9	29	52	76	103	132	162	196	231	251	275	317

☆ Plot the Falcon 9 velocity and time data on your plot of the Apollo 11 velocity vs time plot.



[1] Use this Space X clip (14:30 – 18:30): https://www.youtube.com/watch?v=lSRXacd8wU8

4. Calculate the approximate rate of acceleration for Falcon 9 between 45 to 50 seconds and then between 55 to 60 seconds. What can you say when comparing these two periods during the rocket's acceleration?
   Acceleration isn't constant between 45 and 60 seconds. At 45 seconds the rocket is 'throttling down' in preparation for the Max Q stage. This is the point where the rocket experiences the largest amount of mechanical stress. Between 45 to 50 seconds, the acceleration was 4 m/s^2, while between 55 to 60 seconds the rocket experiences the highest rate of acceleration at 8.4 m/s^2

5. Which of the two rockets, Apollo11 and Falcon 9 has the greater rate of acceleration?
   Falcon 9 – evident in the graph (other than the period between 45 seconds and 55 seconds during throttling down). Determined from the gradient or slope of the graph.

The Payload contents of a rocket vary depending on the mission. The International Space Station for example, required around 40 missions transporting items ranging from crew living quarters to food and toiletries.

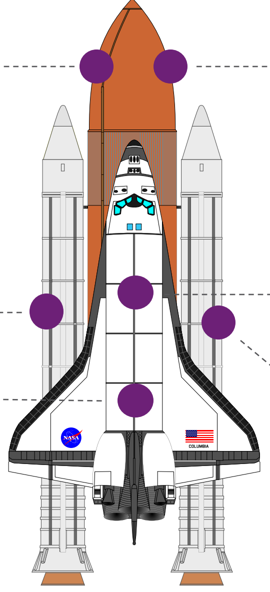
Payload costs have reduced over the years from around $65 000[2] per kilogram during the space shuttle era, to around $2500 per kilogram using SpaceX's Falcon Heavy rocket. The payload ratio of a rocket is the maximum payload of the spacecraft divided by the weight of the propellant and non-fuelled spacecraft, and is given the symbol lambda (λ) where:

md = payload mass
mp = total mass of propellant (fuel)
ms = mass of the spacecraft including empty tanks and boosters (not including payload mass and propellant mass)

$$\lambda = md / (mp + ms)$$

To express this as a percentage, you need to multiply by 100.

✯ Use the following information to determine the approximate payload mass for a space shuttle mission to Low-Earth Orbit (LEO) if the shuttle has a payload ratio of 1.25%



0.0125 = md / [(718 944 + 32 000 + 498 952x2) + 35 426 + 78 000 + 83 915x2)]
0.0125 = md / (1 748 848 + 281 256)
md = 0.0125 x 2 030 104
= 25 376 kg or 25.4 tonnes

https://publicdomainvectors.org/ photos/Space-Shuttle.png
[2] https://ourworldindata.org/grapher/cost-space-launches-low-earth-orbit

As of early 2023 SpaceX's Falcon 9 rocket used on Falcon Heavy is the most powerful rocket in use. As a result of advances in materials and rocket technology along with SpaceX's ability to recover and reuse the Falcon 9 rockets, they are able to offer a very competitive launch service to space.

✯ How does Falcon 9's payload ratio for a LEO launch compare with the approximate payload ratio of 1.25% for a shuttle launch?

Falcon 9 statistics:

ms = 72 300 kg
mp = 1 338 500 kg
md = 63 800 kg

λ = 63 800 / (1 338 500 + 72 300)
= 0.045
λ% = 0.045 x 100 = 4.5%

✯ In 2018 SpaceX launched a car into space. Assuming the car, a Tesla Roadster (1235kg) launched aboard a Falcon Heavy rocket wasn't modified in any way and the Starman mannequin added 40kg to the cars weight. What percentage of the payload mass was the car? And how many Tesla Roadsters with mannequins could they have launched?

The launched Tesla Roadster weight = 1235 kg + mannequin 40kg
= 1275 kg
Percentage of payload mass = 1275 kg / 63 800 kg x 100
= 2.0%
Number of Tesla Roadsters able to be launched = 63 800 kg / 1355 kg
= 50

✯ If the Falcon Heavy was to be used for a mission to the Moon the payload mass would reduce to around 20 000 kg due to the extra fuel needed. If 2.5% of this was taken up with seating for 10 astronauts and the average weight per astronaut was 78.5 kg. What would be the remaining payload capacity if 10 astronauts were on a Moon mission?

Payload mass 20 000 kg x 0.025 = 500 kg total weight of seats
Weight of seats 500 kg + 785 kg (10 astronauts @78.5 kg) = 1285 kg
Remaining payload capacity = 20 000 kg – 1285 kg
= 18 715 kg

With a higher payload ratio than many of their competitors and the ability to reuse the first stage booster of the Falcon 9 rocket SpaceX have an advantage over many of their competitors. Safely landing the booster required numerous attempts until the first successful landing in 2015 (https://www.youtube.com/watch?v=p9FzWPObsWA)

✯ **Extension question** If the first stage booster rocket was released at a height of 80km and fell vertically to Earth. Approximately how long would it take the booster rocket to reach the earth's surface, if the boosters freefall terminal velocity was 450 m/s and the booster accelerated at 9.8 ms2 until it reached terminal velocity?

NOTE: A number of assumptions have been made in this calculation including a terminal velocity of 450 m/s

STEP 1 – Determine time taken to reach terminal velocity using v = u + at

Where: v = final velocity (in this case, 450 m/s)
t = time taken
a = acceleration (9.8 m/s^2)
u = initial velocity (0 m/s since we are starting from rest)

v = u+ at
450 = 0 + 9.8t
t = 450/9.8 = 46 s

STEP 2 – Determine distance travelled up to reaching terminal velocity using $d_{initial} = ½ at^2$

Where: d = initial distance (m)

$d = ½ \times 9.8 \times 46^2$

$d = 10\,386.4$ m

STEP 3 – Determine distance travelled at terminal velocity $d_{terminal}$: $d_{total} – d_{initial}$

$d_{terminal} = 80\,000 – 10\,368.4$

$= 69\,631.6$ m

STEP 4 – Determine time taken to travel 69 631.6m at 450 m/s using $t = d / v$

$t = 69\,631.6 / 450$

$= 154.7$ seconds

∴ Total time taken for the booster to reach the surface = time during acceleration + time during terminal velocity $t_{total} = 46 + 154.7$

$= 200.7$ s or 3.35 minutes

Australian Curriculum

Mathematics

Recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers using digital tools (AC9M9N01)

Apply the exponent laws to numerical expressions with integer exponents and extend to variables (AC9M9A01)

Use mathematical modelling to solve applied problems involving change including financial contexts; formulate problems, choosing to use either linear or quadratic functions; interpret solutions in terms of the situation; evaluate the model and report methods and findings (AC9M9A05)

Solve problems involving very small and very large measurements, time scales and intervals expressed in scientific notation (AC9M9M02)

Use mathematical modelling to solve practical problems involving direct proportion, rates, ratio and scale, including financial contexts; formulate the problems and interpret solutions in terms of the situation; evaluate the model and report methods and findings (AC9M9M05)

Choose appropriate forms of display or visualisation for a given type of data; justify selections and interpret displays for a given context (AC9M9ST04)

Science

Investigate Newton's laws of motion and quantitatively analyse the relationship between force, mass and acceleration of objects (AC9S10U05)